Lab 9: Orbits

Orbital mechanics are the dominant control on the architecture of the solar system. Understanding the orbits of planets, satellites, and smaller bodies explains many things that we observe, including locations of the asteroids and comets, planetary rings, and volcanically active satellites (we'll be looking at these next week).

Remember to make sure that you are using consistent units. Some of the values are provide are in mks (meters-kilograms-seconds) while others are cgs (centimeters-grams-seconds). I didn't do this solely to make things difficult, but to drive in the point that scientists don't always use the same units, so you have to pay attention.

1. Orbits

The year is 3002 and the spaceship *Planet Express* has arrived in the Decapod system. You observe one planet orbiting the system's main star at a distance of .72 AU once every 220 Earth days. Newton's Law of Universal Gravitation and the formula for centrifugal force are, respectively:

$$F_g = \frac{GmM}{r^2} \qquad F_{centrifugal} = \frac{mV^2}{r}$$

(where G Is the gravitational constant (6.67e-11 $m^3kg^{-1}s^{-2}$), m and M are the masses of the planet and star, and V is the orbital velocity of the planet)

a) Draw a top-down sketch. Be sure to indicate m, M, r, and the directions of V, gravitational force, and centrifugal force.

b) Set the forces equal to eachother and solve for M to calculate the mass of the star.

2. Tides

For a uniform, fluid body, the *equilibrium tide* is given by

$$H = \frac{5}{2} R \frac{m}{M} \left(\frac{R}{a}\right)^3$$

where M is the body mass, m is the mass of the tide raising body, R is the body radius, and a is the semi major axis.

a) Calculate the equilibrium tide that the Moon raises on the Earth. The Moon's mass is 7.3e22 kg and it orbits the Earth at 384e3 km. The Earth's radius is 6370 km and has a mass of 6.0e24 kg. What is the equilibrium tide on the Earth? Does your value make sense?

b) Now calculate the tide that the Sun raises on the Earth. The Sun has a mass 333000 times greater than the Earth. Comment on your value…is this surprising?

c) Suppose the Moon was to be twice as far from the Earth as it is today. Ignoring the effect of the Sun, what would be the equilibrium level of the tides?

d) What if the Moon was twice as close?

e) By bouncing lasers off of mirrors left behind by the Apollo astronauts, we know that the Moon is currently moving away at 3.8 cm per year. For extra credit, comment on whether or not this speed could have always been the case (compare it to the age of the Earth-Moon system).

3. Roche Limit

The Roche limit is the distance from a planet at which a moon would be broken apart by the planet's tidal forces. Here you will derive an expression for this limit by making a few simplifying assumptions. We will be looking at a planet-moon system where the moon is **tidally locked**, with the following variables defined:

R: radius of planet *r*: radius of satellite (moon) *M*: mass of planet *m*: mass of satellite

u: mass of rock on satellite near side *D*: distance between the center of m_p and m_s

T: orbital period (time it takes to complete an orbit)

If D is below a certain size (aka the Roche limit), the moon will begin to be ripped to pieces by the planet's tidal forces, so were going to find an expression for this minimum value of *D*. The strategy will be to consider at what *D* a rock of mass *u* on the moon's near-side surface (the side facing the larger body) will be lifted off. In other words, you will balance the forces of gravity that the rock is experiencing from both the planet and the satellite.

a) Draw a picture of this scenario. Include the planet, moon, and rock of interest. Label all variables.

b) Using Newton's Law of Universal Gravitation (see above), what is the gravitational force that the rock (on the moon's surface) experiences from the *moon*?

c) And what is the gravitational force that the rock experiences from the *planet* (remember that the rock is located on the *surface* of the moon)?

d) Imagine another rock, also of mass *u*, is at the *center* of the moon. What is the force of gravity it feels from the planet?

e) Now subtract (d) from (c) (you'll need a common denominator) and simplify to obtain an expression for the tidal force F_T in terms of *G*, *M*, *u*, *r*, and *D*. Make sure to FOIL out the binomials.

f) This expression is messy, but since it's safe to assume that $D >> r$ you can set every *r* in the denominator and every *r* with an exponent in the numerator to zero to obtain an approximation (this will only work if there aren't binomials). Simplify so that you are left with an expression for F_T in terms of *G*, *M*, *u*, *r*, and *D*.

g) The Roche limit is the point at which the rock is feeling equivalent forces from the tidal forces and the force of gravity from the planet (i.e. when $F_G = F_T$), so set you expression from (f) equal to your expression from part (b) and solve for D in terms of r, M, and m.

h) Clean this up by expressing the masses as $\rho * V$ so your expression Is in terms of ρ_p (density of planet), ρ_s (density of satellite), and R (This is an approximation, so it may appear a bit different than versions you may find online. Come see me if you have questions)

i) Check your units. What happens to the Roche limit when ρ_p, ρ_s, and R increase? Does this make physical sense?

j) What you have derived is an approximation for the Roche limit of a *rigid* body. The derivation for a fluid satellite is considerably more complicated, but thinking about it conceptually: do you think that the Roche limit would be further or closer to the planet if the satellite was fluid?

k) Now let's try plugging in numbers. First the Earth-Moon system. The density of the Moon is about ~.6 that of the Earth (why do you think it is smaller than the Earth's density?). If the Moon was moving towards the Earth (it isn't…see question 2) what is the closest the Moon could get to the Earth?

l) The radius of Mars is 3395km and the densities of Mars, Phobos (orbiting Mars at 9300 km), and Deimos (orbiting Mars at 23500 km) are 3.9g/cc, 1.9 g/cc, and 1.5 g/cc. Calculate the Roche limit for both moons. Are Phobos and Deimos stable?

m) On February 15th, 2013, a large meteor broke apart above Chelyabinsk, Russia (recall all the dashboard camera videos on YouTube). Later in the day, a much larger asteroid named 2012 DA14 (diameter ~45m and mass ~1.3e8kg) flew within 27,700km of Earth. Some people (including news anchors on CNN) speculated that the meteor that struck Russia were fragments of 2012 DA14 that were ripped off by Earth's tidal forces. Could this have been the case?

n) Saturn has a mean density of 0.687 g/cm^3 and radius 60000 km. Using densities of .35, .7, and 1 g/cm^3 for the mass of the ring material and the Wikipedia (https://en.wikipedia.org/wiki/Rings_of_Saturn#Major_subdivisions_of_the_rings) for the actual ring sizes, test if Saturn's rings formed as the result of a moon being broken up by falling within the Roche limit. (for simplicity you can assume Saturn is solid, but comment on what effect this assumption has in your answer)