**Planetary Geology Lab 8-Small bodies (asteroids and comets)**

Small bodies in the solar system include comets, asteroids, Kuiper Belt objects, and the Oort cloud. Many of these objects are thought to be the leftovers from planetary accretion, and can therefore offer clues into the early history of the solar system.

Technically speaking, comets contain volatiles while asteroids do not. However, the line between these two types of bodies is very blurred: a body that contains ice at its core, but not near the surface, will appear (and be labeled) as an asteroid, while a comet that has lost all of its accessible volatiles will be appear to be an asteroid.

In the two problems here you will explore the shape and interior composition of asteroids. A couple of these questions involve a little calculus. If you're having trouble come see me.

**Problem 1**

Many small bodies differ from the planets in that they are not spherical. Planetary bodies become spherical when the force of gravity is able to overcome the planet's strength. Here we will investigate how large an asteroid needs to be in order to become spherical.

a) The acceleration of gravity on a planetary body is given by the equation

$$g = \frac{GM}{r^2}$$

where $G$ is the gravitational constant ($6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$), $M$ is the mass, and $r$ is the radial distance from the center of that mass. Write down an expression for the gravity inside a planet of uniform density $\rho$, in terms of $G$, $\pi$, $r$, $\rho$.

b) Recall from the isostasy lab that the pressure $P$ is given by the generalized hydrostatic assumption

$$\frac{dP}{dr} = -\rho g$$

Combine this with your expression from part (a) to obtain a generalized expression for the pressure as it changes with depth ($dP/dr$) in terms of $G$, $\pi$, $r$, and $\rho$.

c) Now integrate $P$ with respect to $r$ to obtain a general expression for the pressure P(r) inside a planet in terms of $G$, $\pi$, $r$, $\rho$, and constant $c$.

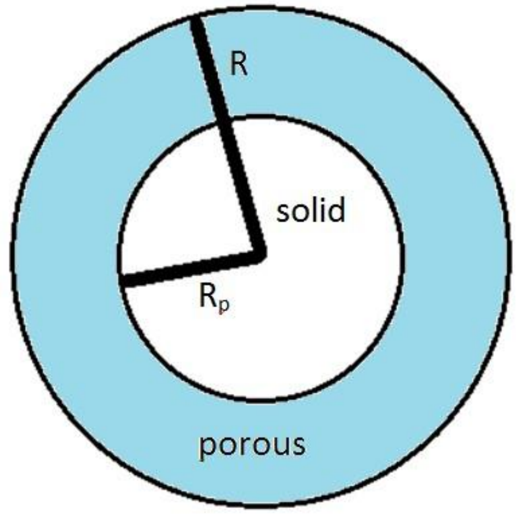
d) We know that the pressure at the surface of the body is zero ($P=0$ when $r=R$). Use this to find the value of the constant c.

e) Now write down an expression for the pressure at the center of the planet ($r=0$) in terms of $G$, $\pi$, $R$, and $\rho$.

f) If geologic materials start to flow at a pressure of roughly 10 MPa (mega-pascals), how big can a planet of density 4000 $kgm^{-3}$ get without assuming a spherical shape?

g) Would you expect most asteroids to be spherical or not? Do some quick research online (Wikipedia) on the typical sizes and shapes of asteroids. Do observations confirm or contradict your estimates (remember, since this is a rough estimate anything within an order of magnitude counts as a confirmation of your results!)

**Problem 2**




Some asteroids are monoliths (solid rocks), while others are rubble piles, held together solely by their own gravity. Others are some combination of these two extremes, with a solid interior and a loose, unconsolidated mantle. Here we will look at the interior structure of an asteroid which has a porous near-surface and a solid interior (see figure).

The porosity (fraction occupied by voids) of the surficial layer is $\phi$, the density of the interior solid portion is $\rho$, and the density of the nearsurface layer is $(1-\phi)\rho$.

a) Write down an expression for the mass of the core, in terms of $R_p$ and $\rho$.

b) Now an expression for the mass of the shell, in terms of $R$, $R_p$, $\phi$, and $\rho$.

c) Now an expression for the overall (mean) density of the asteroid, in terms of $R$, $R_p$, $\phi$, and $\rho$.

d) What happens to the mean density from your expression if $R_p$=0, $R_p$=$R$ or $\phi$=0? Do these behaviors make physical sense?

e) Using the equation for gravitational acceleration from problem (1) write down an expression for the gravitational acceleration $g$ at the bottom of the porous layer (i.e. just outside the solid core) in terms of $G$, $R_p$, and $\rho$.

f) The pressure exerted by a column of height $h$ and density $\rho$ is given by the hydrostatic pressure equation $P = \rho g h$. Assuming that $g$ is constant throughout the porous layer, write down an expression for the pressure at the base of the porous layer in terms of $G$, $R$, $R_p$, $\phi$, and $\rho$.

g) Now were going to differentiate to find an expression for the value of $R_p$ which **maximizes** this pressure for a **given value** of $R$, $\phi$, and $\rho$. To do this, differentiate the expression you got for $P$ from part (f) with respect to $R_p$, set it to zero, then solve for $R_p$)

h) Write down an expression for this maximum pressure (plug what you got for $R_p$ into $R$ for the equation you derived in part (f)

i) Pores will start to close if this maximum pressure exceeds roughly 1 MPa. Assuming that $\phi$=0.3, $\rho$=3000 kg m$^{-3}$, how big can an asteroid get before pore closure starts?