Lab 10: Tidal heating

Recall that:
Elliptical orbits (Kepler's Laws) are explained by Newton's inverse square law for gravity
Tides arise as gravitational attraction varies across a body
Tides can rip a body apart if it gets too close to its primary (the Roche limit)

Here we will examine the effects that tides have on moons in the outer solar system. Tidal motions are expressed as bulges on a planet or satellite, and the stress and friction caused by these bulges generate heat. This heat is what makes the outer planet satellites (including Io, Europa, Enceladus, and Titan) active.

1. Heat measurements on Io

   Io (radius 1820 km and mass 8.93×10^{22}kg) is the innermost of the Galilean moons. The Galileo spacecraft measured the surface heat flux of Io to be 3 Wm^{-2}.

   a. What is Io's surface area?

   b. What is the total heat (in watts) being emitted from Io's surface?

   c. So how much heat is being produced per kilogram in the interior?

   d. Much of the heat in the terrestrial planets (including Earth) comes from the decay of radioactive elements, such as ^{26}Al. Typical solar system rocks (chondrites) produce heat at a rate of about $\sim 4 \times 10^{-12}$ W kg^{-1}. If Io's bulk composition is chondritic then what fraction of Io's heat comes from radioactivity rather than tides?

   e. for simplicity we have assumed that Io does not have an iron core….how would you expect this to affect your estimate?

2. Heating on Europa

   Europa has a radius of 1560 km, mass of 4.8×10^{22} kg, and a water layer ~150km thick that is topped by an ice shell.

   a. Using the heat flux equation

   $$q = k\frac{dT}{dz}$$

   and assuming a thermal conductivity (k) of 3 $Wm^{-1}K^{-1}$ (ice is 2 and water is 4), surface temperature of 100K, and an ice shell thickness of ~4 km, what is Europa's heat flux?

   b. How much radiogenic heat is produced in the rocky portion of Europa via radioactive decay (once again, assume that there is no iron core)? What fraction of Europa's heat comes from radioactivity rather than tides?

   c. If there were no tidal heating on Europa the how thick would the ice-shell be?

   d. How thick would the ice shell be on Ganymede if the rocky portion of that body produces radiogenic heat at the chondritic rate and the surface temperature is similar to Europa? (Europa's H2O layer is ~150km thick and the radius of Ganymede's rocky interior is about 68% of the body.)

3. Tidal heating on Enceladus

Now let's determine the approximate heating to do tidal friction on Enceladus. Recall that for a uniform, fluid body the equilibrium tide H is given by

$$H = \frac{5}{2} R \frac{m}{M} \left(\frac{R}{a}\right)^3$$

where M and R are the body's mass and radius, m is the mass of the tide-raising body, and a is the semimajor axis

a. The magnitude of the *diurnal* tide felt by the satellite is $3e$ times the equilibrium tide, where e is the eccentricity. Write down an expression for the magnitude of the satellite diurnal tide.

b. The tidal strain (ε) is simply the diurnal tide divided by R. Write down an expression for the tidal strain.

c. The strain rate is the tidal strain divided by the orbital period. Using equations derived from Kepler's Laws of Planetary Motion,

$T = \frac{2\pi}{\omega}$ and $\omega^2 = \frac{Gm}{a^3}$ (where T is orbital period and ω is angular frequency)

determine the period and thus write down an expression for the strain rate.

d. Assuming that the satellite is a viscous fluid, the heating rate per unit volume is given by viscosity x strain rate^2. Write down the heating rate per unit volume in terms of *e, m, M, R, a, G,* and the viscosity η.

e. The heating rate per volume should have units of Wm^{-3}. Check your units to make sure the equation you derived in part (d) makes sense. (Viscosity has units of [Pa s] and G has units of [$m^3kg^{-1}s^{-2}$])

f. Now find the total heating rate in the satellite in terms of *e, m, M, R, a, G* and η.

g. What happens to this heating rate if *e* decreases or *a* increases? Does this make sense?

h. Friction results in a lag of the tidal bulge behind the point on the body's surface where the gravitational force is strongest, and this means that only some of the energy gets turned into heat. This lag (*f*) needs to be multiplied by the total heating. Using f=0.01, calculate how much tidal heat (in W) Enceladus should generate, using the following parameters: $M=1\text{x}10^{20}$ kg, $m=5.7\text{x}10^{26}$ kg, R=250 km, $a=2.4\text{x}10^5$ km, $\eta=10^{13}$ Pa s (this is the value for warm ice), e=0.0045 and G=6.67x10-11 $m^3kg^{-1}s^{-2}$

i. Spacecraft observations indicate that the South pole of Enceladus has a surface heat flux of roughly 0.2 Wm-2. Comment on this value in view of your answer to (h).